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STRUCTURALLY-HYPERBOLIC ALGEBRAS DUAL TO THE CAYLEY-DICKSON AND CLIFFORD ALGEBRAS

OR NESTED SNAKES BITE THEIR TAILS

DIANE G. DEMERS

For Elaine Yaw in honor of friendship

ABSTRACT. The imaginary unit i of $\mathbb{C}$, the complex numbers, squares to -1; while the imaginary unit j of $\mathbb{D}$, the *double number*s (also called *dual* or *split complex numbers*), squares to $+1$. L.H. Kauffman expresses the double number product in terms of the complex number product and vice-versa with two, formally identical, dualizing formulas. The usual sequence of (structurally-elliptic) Cayley-Dickson algebras is $\mathbb{R}, \mathbb{C}, \mathbb{H}, ...$, of which Hamilton's quaternions $\mathbb{H}$ generalize to the split quaternions $\mathbb{H}\!\!\!\setminus$. Kauffman's expressions are the key to recursively defining the dual sequence of structurally-hyperbolic Cayley-Dickson algebras, $\mathbb{R}, \mathbb{D}, \mathbb{M}, ...$, of which Macfarlane's hyperbolic quaternions $\mathbb{M}$ generalize to the split hyperbolic quaternions $\mathbb{M}\!\!\!\setminus$. Previously, the structurally-hyperbolic Cayley-Dickson algebras were defined by simply inverting the signs of the squares of the imaginary units of the structurally-elliptic Cayley-Dickson algebras from -1 to $+1$. Using the dual algebras $\mathbb{C}$, $\mathbb{D}$, $\mathbb{H}$, $\mathbb{H}\!\!\!\setminus$, $\mathbb{M}$, $\mathbb{M}\!\!\!\setminus$, and their further generalizations, we classify the Clifford algebras and their dual *orientation congruent* algebras (Clifford-like, noncommutative Jordan algebras with physical applications) by their representations as tensor products of algebras.

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